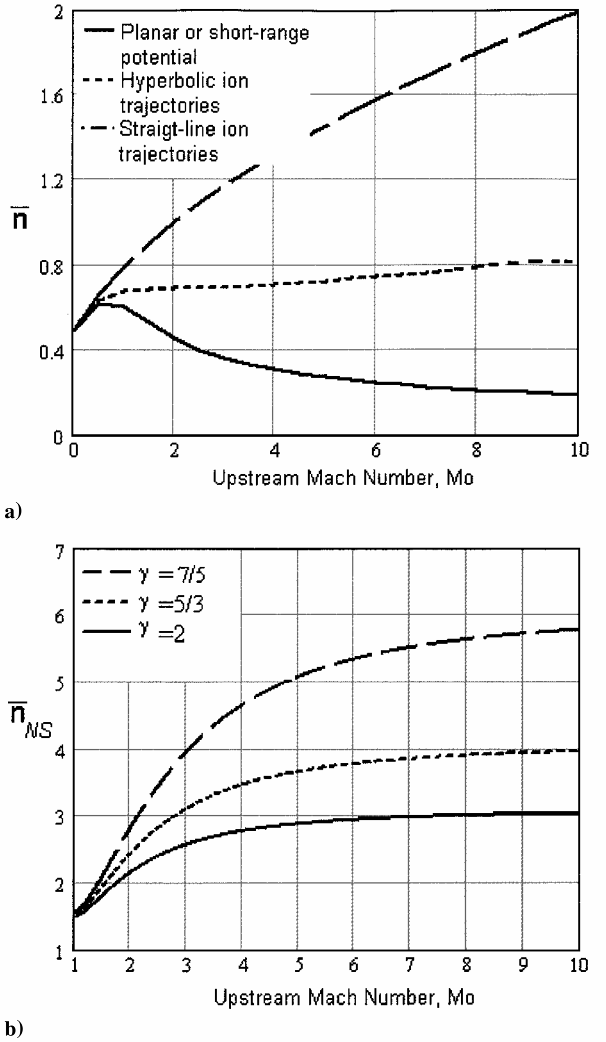

Fig. 4 Density ratio at the stagnation point ($\chi = M_0^2/2$) as a function of a) upstream Mach number M_0 for free molecular flow (collisionless ions) and b) upstream Mach number for continuum, hypersonic flow across a normal shock wave.

shock in this case can be expressed as a function of the upstream Mach number[4]:

$$\bar{n}_{\mathrm{NS}} = \frac{(\gamma+1)M_0^2}{2+(\gamma-1)M_0^2}\left(1+\frac{\gamma-1}{2}\left\{\frac{1+[(\gamma-1)/2]M_0^2}{\gamma M_0^2-(\gamma-1)/2}\right\}\right)^{1/(\gamma-1)} \tag{16}$$

Equation (16) is plotted for different ratios of specific heats in Fig. 4b. By comparison, for a monatomic gas ($\gamma = \frac{5}{3}$) at $M_0 = 7$, the maximum density ratio is 3.86, which is by a factor of three larger than the peak density (1.16) computed by Eq. (15). At the stagnation point, $M_0 = \sqrt{(2\chi)}$, the density does not exceed unity for Mach numbers less than 10, as shown in Fig. 4a.

Conclusions

An integral expression for the density enhancement of hypersonic ions incident on a repulsive potential has been presented for the case of a $1/r^2$ force field, together with expressions for upper and lower limiting cases. For the interesting case of orbital velocity in a 0.1-eV oxygen plasma, deceleration of the incident ions produces a 10% density enhancement, whereas reflected (outgoing) ions contribute another 18%. The absolute upper enhancement for this case (planar potential with reflection) gives a factor of 4.6. The density maximum occurs at a potential of about 4 V, which is less than the ram ion energy of $E/e = 4.7$ V. The modest density enhancements calculated here are compared to density enhancement achieved in shocked hydrodynamic flows for which density ratios can exceed four at comparable Mach numbers.

Acknowledgment

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Rationale for Supersonic Afterburning Rocket Engines

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Nomenclature

g_0 = gravitational acceleration at sea level, ms^{-2}
I_{spvac} = specific impulse (in vacuum), s
m_b = mass at burnout (main engine cutoff), kg
m_e = total main engine mass, kg
m_0 = initial mass (at liftoff), kg
T_b = thrust at burnout (main engine cutoff), N
T_0 = initial thrust (at liftoff), N
ΔV_{tot} = total delta-V requirement to attain orbit
μ = overall engine oxidizer/fuel-mass-flow-rate (mixture) ratio
μ_c = oxidizer/fuel-mass-flow-rate (mixture) ratio in chamber
ρ_{LH} = liquid hydrogen density, kg m^{-3}
ρ_{LOx} = liquid oxygen density, kg m^{-3}
ρ_p = bulk propellant density, kg m^{-3}
τ_e = main engine thrust-to-weight ratio, $T_0/(m_e g_0)$

Introduction

THE introduction of low-cost, commercial, reusable space transportation systems depends heavily on the appropriate

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utilization and/or development of propulsion systems. In particular, the viability of single-stage-to-orbit (SSTO) vehicles probably hinges on new liquid rocket engine developments that push back existing performance limits. This Note outlines the rationale for further study of a new class of thrust-augmented-nozzle (afterburning) engines that may fulfill SSTO propulsion requirements.[1]

SSTO Propulsion Requirements

To attain SSTO the initial (gross) mass over burnout mass of a rocket using liquid oxygen (LOx) and liquid hydrogen (LH) propellants would have to be at least $m_0/m_b \cong 8$, assuming a baseline vacuum specific impulse of about $I_{\text{spvac}} \cong 455$ s and ascent from sea level at the equator to a zero-inclination Earth orbit of about 200-km altitude, according to the well-known ideal rocket equation

$$\Delta V_{\text{tot}} = g_0 I_{\text{spvac}} \ln\{m_0/m_b\} \tag{1}$$

During vertical liftoff (initially) the optimum design thrust-to-weight ratio for such an SSTO vehicle would typically be about $T_0/(m_0 g_0) \cong 1.3$–1.4. To reduce structural loads most studies assume a maximum burnout acceleration of about 3–$4g_0$, so that the burnout thrust over initial thrust must be limited to $T_b/T_0 \cong 0.4$. If the main ascent engines have a sea level thrust to weight of about $\tau_e = T_0/(m_e g_0) \cong 50$ (a value based on contemporary high-pressure LOx–LH engine technology), then the total engine mass must constitute about one-fifth of the burnout mass, i.e., $m_e/m_b \cong 0.2$, or about one quarter of the dry vehicle mass.

The payload fraction of such an SSTO vehicle is likely to be marginal (0–2% of initial mass) if it is to be fully reusable.[1] Hence, on cursory inspection of the ideal rocket equation (1), it appears essential to prioritize high vacuum specific impulse ($I_{\text{spvac}} > 455$ s) to increase the burnout mass fraction. However, only small increases in LOx–LH engine specific impulse can actually be achieved by specifying higher chamber pressures (etc.), and it may be more beneficial to focus on other engine performance parameters. In particular, it may be more beneficial to reduce the main propulsion system mass. The relative trade between main engine mass and vacuum specific impulse is easily appreciated when it is noted that a specific impulse fall of about 1 s ($I_{\text{spvac}} \rightarrow 454$ s) could be balanced by a 2% improvement in main engine thrust to weight ($\tau_e \rightarrow 51$) at the design point specified earlier. Similarly a 10-s fall in specific impulse ($I_{\text{spvac}} \rightarrow 444$ s) could be balanced by a 30% improvement in thrust to weight ($\tau_e \rightarrow 65$). Therefore reductions in LOx–LH engine I_{spvac} could be tolerated and might even be advantageous for SSTO vehicles, provided that the main propulsion system is made to be sufficiently lightweight.

Previous SSTO studies[1–5] have also recognized the need to reduce propellant volumes to reduce vehicle size and dry mass. The propellant volume is dictated by the volumetric specific impulse, $\rho_p I_{\text{sp}}$, where the propellant density is given by

$$\rho_p = \rho_{\text{LH}} \rho_{\text{LOx}} \{1 + \mu\} / \{\rho_{\text{LOx}} + \mu \rho_{\text{LH}}\} \tag{2}$$

where μ is the overall mixture ratio of the propellants flowing to the engine(s). Early in the vehicle ascent trajectory it may be beneficial to increase volumetric specific impulse, but a shift to high LOx–LH mixture ratios ($\mu > 8$) has not often been advocated essentially because of engine cooling problems. Instead, propellant volume reduction has been achieved by advocating dual fuel options,[2–5] e.g., the use of a hydrocarbon (burning with LOx and replacing low-density LH, thereby reducing propellant volume), although these options obviously increase propulsion system complexity. If a way could be found to utilize high-LOx–LH mixture ratios early in the ascent to increase bulk propellant density ρ_p, without cooling problems, then an increase in payload fraction might be achieved more simply.

Note that the selection of nozzle expansion ratio for SSTO vehicles is generally a compromise between overexpansion at sea level and underexpansion in vacuum, unless extendable nozzle mechanisms are utilized. In previous SSTO studies,[1–5] progressive shutdown of an ensemble of separate engines is generally assumed with two different nozzle expansion ratios to achieve the required in-flight thrust reduction of about 3:1. If it were possible to increase nozzle expansion ratio (to increase I_{spvac}) without causing overexpansion at low altitude, as well as providing some simple means of in-flight thrust reduction, then an overall system advantage might be achieved.

One possible concept to realize these propulsive requirements appears to be a class of Supersonic After-Burning Rocket Engine (SABRE) cycles[1,6] that involve injection of propellants into the nozzle in order to augment thrust by a supersonic combustion process.

Supersonic Afterburning in Nozzle

A "classical" rocket engine essentially involves combustion within the main chamber and expansion through a convergent–divergent (bell-shaped) nozzle where the flow is supersonic in the divergent portion. Whereas most contemporary cycles involve preburning to drive turbopumps, the dominant chemical energy release occurs subsonically in a single combustion phase within the main chamber. In high-pressure LOx–LH engines a chamber injection mixture ratio of about $\mu_c = 5$–7 (fuel rich) is selected to achieve high specific impulse and to avoid excessive throat heating where the flow is sonic. Nozzle flows are essentially in equilibrium during expansion, resulting in near optimal specific impulse when the nozzle exit and ambient pressures are equal. However, useful chemical energy is wasted: assuming that $\mu_c = 5$ is used in the main chamber (an equivalence ratio of 1.6), at a chamber pressure of 25 MPa, about 6% of the expanding flow comprises unburned hydrogen gas. At a station inside the nozzle where the expansion ratio is 6, the Mach number, pressure, and temperature are about 2.9, 0.6 MPa, and 1800 K, respectively. These flow conditions appear to be suitable for mixing controlled supersonic combustion[7] by direct injection of LOx into the nozzle, i.e., suitable to form a supersonic afterburner (Fig. 1).

Two points should be noted immediately. First, injection of secondary propellants into nozzles for thrust vector control (or just for cooling purposes) is well known,[4] but previous studies have generally assumed injection of fuel or inert fluid into already fuel-rich combustion products; hence afterburning (reheat) in the nozzle was not intended and could not occur. Second, studies of mixing controlled supersonic combustion have generally assumed the injection of fuel into a supersonic airflow.[7] In the SABRE cycles described herein air is not required and is relevant only to the external flow over the vehicle.

Direct LOx injection into the nozzle flow is likely to result in shock losses caused by the interference of the injector(s) with the main nozzle flow, as well as pressure losses caused by the mixing-combustion process, resulting in a specific impulse penalty. Nevertheless, the resulting chemical energy release coupled with the increased nozzle mass flow should arguably result in significant augmentation of thrust.

To create near-stoichiometric conditions with the unburned hydrogen gas, LOx injection would have to be limited to about 50% of the main flow (by mass, when $\mu_c = 5$), and hence with such a scheme the thrust augmentation factor would similarly be limited to about 50%. If, however, a specific impulse reduction is planned over an early flight phase (to increase $\rho_p I_{\text{sp}}$), then more LOx could be injected to achieve higher thrust augmentation factors. The nozzle would also be extended (or enlarged) for the increased mass flow, without overexpansion occurring.

With such a SABRE scheme, at best it would be possible to match the performance of a classical engine of the same overall mixture ratio (Fig. 2).[8] For example, when the injected LOx nozzle flow is as large as the main chamber LOx injection, the overall LOx–LH

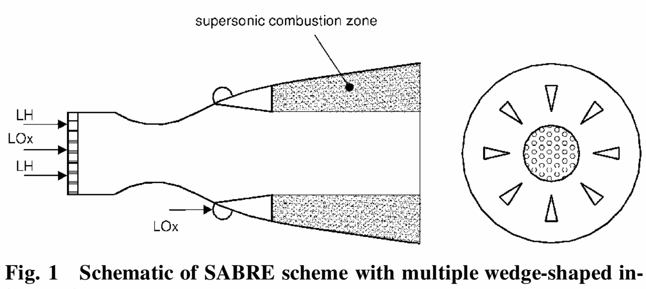

Fig. 1 Schematic of SABRE scheme with multiple wedge-shaped injectors in nozzle.

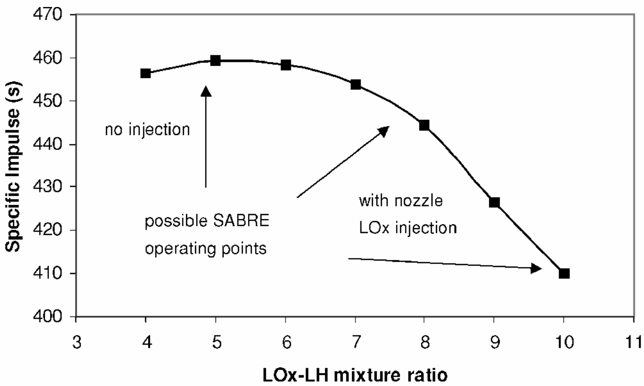

Fig. 2 Variation of ideal vacuum specific impulse of an LOx–LH engine operated at a chamber pressure of 25 MPa and nozzle expansion ratio of 50:1 using the calculation methods of Gordon and McBride.[8]

mixture ratio is doubled, $\mu \rightarrow 10$; hence I_{spvac} would fall below 410 s, but ρ_p would increase by about 50%. When the afterburner is switched off, higher specific impulses would resume at the main chamber LOx–LH mixture ratio $\mu = \mu_c = 5$ and the enlarged nozzle would then result in higher expansion ratios, possibly offsetting losses caused by the shock interference of the nozzle injectors, if they are not mechanically retracted.

The mass of the LOx turbopump required for this particular SABRE cycle might possibly be less than a classical engine of the same sea-level thrust because the pressures at the nozzle injection point are well below the main chamber pressures. Furthermore, the throat cooling problems associated with high-LOx–LH-mixture-ratio engines would not be encountered. One potential disadvantage, however, might be the cooling of the nozzle injectors themselves.

Conclusions

The SABRE scheme presented, sketched schematically in Fig. 1, is just one possible layout. Of course, there are other feasible SABRE configurations. There is also a gamut of alternative SABRE cycles involving different injection schemes (e.g., injection of LH into an LOx-rich nozzle flow, or combined LH and LOx injection, or even injection of turbocompressed air), all of which deserve attention at a conceptual design level. Like the rationale for dual-fuel-propulsion systems,[2–5] the main rationale for these SABRE cycles would be an overall improvement engine thrust to weight at liftoff, improved volumetric specific impulse at low altitude, and use of an enlarged nozzle to permit higher expansion ratios, as well as the possibility of in-flight thrust reduction.

The actual substantiated advantage of these SABRE cycles obviously depends on a more detailed analysis than is presented here. This brief Note has merely set out to present a new concept and to demonstrate that there is indeed a promising rationale for SABRE cycles calling for the further study.

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